ECON 4335 – Economics of Banking: Seminar 2 (week 9)

Emil Cappelen Bjøru and Fredrik Kostøl

Problem 1: Adverse selection

- Entrepreneurs:
  - No initial wealth ($A = 0$), risk-neutral and protected by limited liability
  - May be one of two types: "good" (with probability α) or "bad" (with probability $(1-\alpha)$)
- Investment:
  - Safe: $I = 1$. Undertaken by good entrepreneurs, gross return $\{G, 0\}$ with probability $\{p, 1-p\}$
  - Risky: $I = 1$. Undertaken by bad entrepreneurs, gross return $\{B, 0\}$ with probability $\{q, 1-q\}$

    $1 > p > q > 0, \quad B > G > 0, \quad pG = qB = \mu, \quad m \coloneqq \alpha p + (1-\alpha)q$
- Banks:
  - Monopolist and risk-neutral
  - Funding cost: $1 + f$, but $f = 0$

a) Full Information

$V_G^L = p(1 + i^G) - 1$: The expected return of the bank when lending to a good borrower

$V_B^L = q(1 + i^B) - 1$: The expected return of the bank when lending to a bad borrower

$V_G^e = p(G - (1 + i^G))$: The expected return of the good entrepreneur

$V_B^e = q(B - (1 + i^B))$: The expected return of the bad entrepreneur

Since the bank acts as a monopolist, and the entrepreneur is protected by limited liability and has no initial wealth, the monopolist will extract all profits by offering contracts such that $V_B^e = V_G^e = 0$

$V_G^e = 0 \Rightarrow G = (1 + i^G) \Rightarrow i^G = G - 1$

$V_B^e = 0 \Rightarrow B = (1 + i^B) \Rightarrow i^B = B - 1$

No return is left for neither of the entrepreneurs, who in equilibrium will be indifferent between undertaking the projects or not.

The bank receives the entire net present value of the project in both cases:

$V_G^L = p(1 + G - 1) - 1 = pG - 1 = \mu - 1$

$V_B^L = q(1 + B - 1) - 1 = qB - 1 = \mu - 1$

Because the bank is risk-neutral it doesn't matter whether it makes contracts with good or bad borrowers, as it only cares about the expected return of the investment, not the variance.

b) Asymmetric Information

The bank can no longer distinguish the two types before writing the contract. We will assume in the following that it cannot write state-contingent contracts with verifiable outcome, i.e. contracts on the form $r = \{0, r^G, r^B\}$ contingent on the state-of-the-world being respectively $0, G\ or\ B$.

Since the bank no longer can distinguish the two types from each other, it can neither write type-dependent contracts. It will now offer contracts with interest rate r to both types.

Let PC_i denote the participation constraint for type i. If the constraint is violated, type i will choose not to participate, i.e. not to undertake the project.

$$PC_G: \quad V_G^e = p(G - (1 + r) \geq 0 \Leftrightarrow G \geq 1 + r$$

$$PC_B: \quad V_B^e = q\big(B - (1 + r)\big) \geq 0 \Leftrightarrow B \geq 1 + r$$

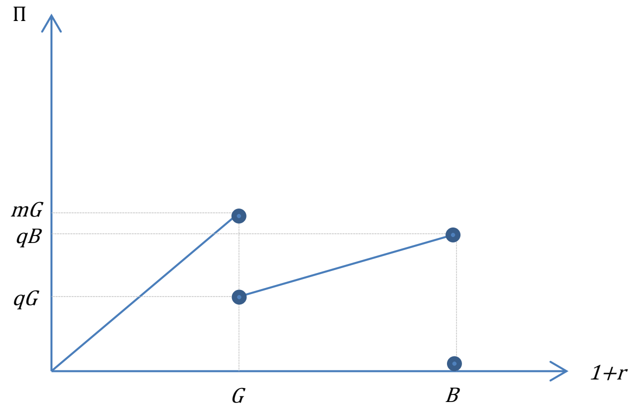
Hence, when:

$$(1 + r)\epsilon \begin{cases} [0, G], & both\ good\ and\ bad\ types\ will\ apply\ for\ loans \\ [G, B], & only\ bad\ types\ will\ apply\ for\ loans \\ [B, \infty), & neither\ good\ nor\ bad\ types\ will\ apply\ for\ loans \end{cases}$$

Let Π denote the bank's expected gross return:

- $(1 + r)\epsilon\ [0, G] \quad \Rightarrow \Pi = [\alpha p + (1 - \alpha)q](1 + r)I = m(1 + r)$, and $\frac{d\Pi}{dr} = m$
- $(1 + r) = G \quad \Rightarrow \Pi = mG$
- $(1 + r)\ \epsilon\ [G, B] \quad \Rightarrow \Pi = q(1 + r)$, and $\frac{d\Pi}{dr} = q$

Since p is greater than q, it follows that a weighted average of p and q also must be greater than q. Hence, when *(1+r)* goes from G to $G + \varepsilon$ for some infinitesimal ε, there is a drop in the bank's expected gross return and in the marginal gross return of an increase in the interest rate. This is illustrated in figure 1 below.



Figur 1

c) Optimal interest rate

If $mG > qB$, then $1 + r = G$ is optimal. If $mG < qB$, then $1 + r = B$ is optimal. It is never optimal to be in the intervals $[0, G)$ or $(G, B]$.

$$mG > qB \Leftrightarrow [\alpha p + (1-\alpha)q]G > qB \Leftrightarrow \alpha > \frac{q}{p-q}\frac{B-G}{G}$$

So, the profit-maximizing interest rate will depend on the fraction of good entrepreneurs in the economy.

Problem 2: Moral Hazard

- Entrepreneurs: Risk neutral, protected by limited liability, no initial assets ($A=0$)
- Investment: $I = 1$. Return depends on an unverifiable action
  - Unverifiable action: Choose safe or risky investment
    - Safe: $\{(p, G); (1-p, 0)\}$
    - Risky: $\{(q, B); (1-q, 0)\}$
    - $B > G, \quad p > q, \quad pG > 1 + i > qB \quad i = 0$
      - Only the safe project has a positive net present value (NPV)
  - Repayment: R

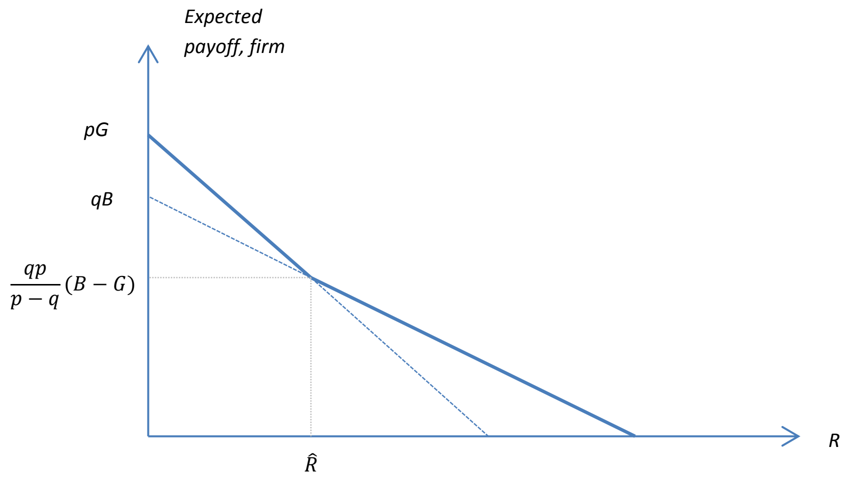
* Moral hazard: The investors cannot observe or verify the action taken by the entrepreneur. The firm may then have an incentive to choose (q, B) and then misreport the actual outcome if the project is a success.

a)

The firm chooses the safe investment if the expected return by doing so is greater than the expected return of choosing the risky investment, i.e.:

$$p(G-R) \geq q(B-R) \Leftrightarrow R \leq \hat{R} \equiv \frac{pG-qB}{p-q}$$

Hence, for $R \leq \hat{R}$, the firm will choose the safe strategy. For $R \geq \hat{R}$, the firm will choose the risky strategy. See figure 2 below



Figur 2

b) No monitoring

Clearly $R \leq \hat{R}$ must hold in a competitive credit market equilibrium. If $R > \hat{R}$, entrepreneurs will choose the risky investment, which has a negative net present value.

In what follows, and in the rest of this problem, IC_j and PC_j denotes the incentive compatibility constraint and the participation constraint for agent j, where $j \,\epsilon\, \{e, i, b\}$ and e, i and b are short for entrepreneur, investor and bank, respectively.

IC_e: $\quad p(G-R) \geq q(B-R) \Rightarrow R \leq \hat{R} = \frac{pG-qB}{p-q}$

PC_i: $\quad pR \geq I = 1$

, where PC_i simply states that in order for investors to be willing to invest in the project, their expected return must be at least as big as the cost.

Since there is no reason to let R be strictly less than $\hat{R}$, IC_e should hold with equality in optimum.

Combining IC_e and PC_i gives:

$(*)$ $\quad p\frac{pG-qB}{p-q}$>1, which must be satisfied in a competitive credit market equilibrium with only direct finance. Define $\bar{p}$ as the p solving $(*)$

d) Monitoring

Banks have the ability to monitor the entrepreneurs. Monitoring prevents the entrepreneur from choosing the risky investment, but incurs a cost c to the bank. This removes the initial problem of moral hazard, since there no longer is any action the entrepreneur can take that affects the outcome of the project. However, a new problem of moral hazard arises. Since the bank must bear the cost of monitoring, the incentives it faces must be structured in a way such that it actually monitors.

Let $I = I_i + I_b = 1$ and $R = R_i + R_b$

The conditions for having competitive equilibrium with a monitoring bank are the following:

IC_b: $pR_b \geq qR_b + c \Leftrightarrow R_b \geq \frac{c}{p-q}$

PC_b: $pR_b \geq I_b + c \Leftrightarrow I_b \leq pR_b - c$

PC_i: $pR_i \geq I_i$

Combining the above constraints with equality in IC_b and PC_b yields:

$$p(R - R_b) \geq I - I_b \Leftrightarrow pR - pR_b \geq 1 - (pR_b - c) \Leftrightarrow pR > 1 + c \Leftrightarrow p \geq \frac{1+c}{R} \equiv \bar{\bar{p}}$$

As b) and c) shows, whether we will have monitoring or not depends on the value of p, as shown in figure 3 below.

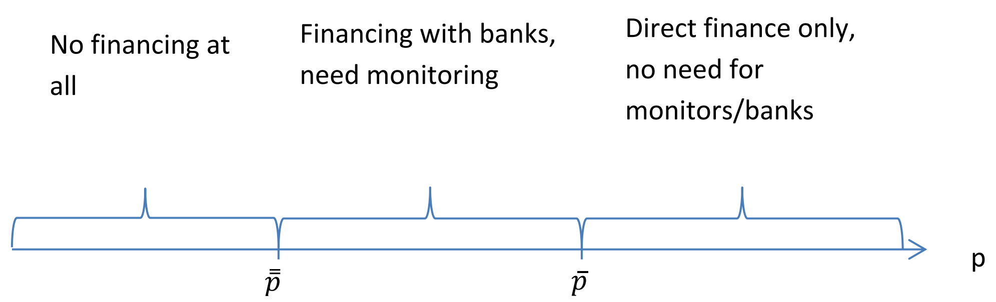


Figure 3